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The Input-Output Model

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N O T A

El presente trabajo del Dr. Angel L. Ruíz Mercado, es la continuación de "A Note on the Relation Between Input-Output and Income and Product Accounting Systems", publicado por la Unidad de Investigaciones Económicas en su Serie de Ensayos y Monografías, Número 36 de febrero de 1984.

El primer trabajo mencionado se dedicó, exclusivamente, al sistema de contabilidad de la técnica de insumo-producto. El presente trabajo introduce el modelo de insumo-producto formalmente.

Próximamente, se publicará un tercer artículo donde se desarrollará el modelo dinámico de insumo-producto.

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THE INPUT-OUTPUT MODEL

by Angel Luis Ruíz, Ph.D.*

Two general equilibrium systems have underlied modern macroeconomic thinking. These are Keynesian economics and Leontief's input-output system. The increasing popularity of these systems can be easily explained. Both deal in a simple way with problems empirically relevant and of great importance from the point of view of policy making. Although static in their original and basic format, both have been starting points for the study of dynamic problems, like economic growth. Keynes' system was originally developed as an alternative to the "classical" economics while Leontief's system was an attempt to make Walrasian concept of general interdependence empirically manageable. Theoretical explorations in the area of economic interdependence began more than two centuries ago with the construction of the Tableau Economique (1766) by the famous French economist Francois Quesnay and culminated with the work of Leon Walras (1874). However, the general equilibrium system of Walras was empirically unworkable. It was not until W. Leontief published the input-output

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system for the U. S. economy[1/], that the Walrasian system was rendered to manageable proportions by constructing a model of interindustrial dependence. Leontief presented the first version of his model in 1936[2/], but set out the static system more fully in his book. The structure of the american economy, 1914-1939[3/]. In discussing Leontief's system it is convenient to distinguish between the "input-output model", which deals with the behavior of the economic system, and the "input-output table of matrix" which is a purely definitional set of relationships that play an important role in modern national accounting systems. The value of the input-output table is indisputable whatever the merits of the model may be.

In a previous work I explained the accounting equations of the system.

The transition toward an analytical system (the input-output model) requires the introduction of certain behavioral assumptions about the economic system. At least three assumptions are made:

1. Identity of the industry and product
2. Homogeneity of the product
3. The existence of fixed coefficients

The first assumption we make is that each industry produces only its own characteristic product and no other: The motor industry only makes motor vehicle. This can be called the assumption of identity and product.

The second assumption we make is that each product is uniform: all motor vehicles are the same. This can be called the assumption of product homogeneity[4/].

1/ W.W. Leontief, "Quantitative Input-Output Relations in the Economic System of the United States", The Review of Economic and Statistics (August, 1936).

2/ Ibid.

3/ W.W. Leontief, The Structure of the American Economy, Oxford University Press, New York (1941).

4/ This assumption involves the so called: "Leontief's production function" which implies constant returns to scale.

The third assumption made is that at any point in time each input is required in a fixed ratio to the output into which it enters, a ratio which is independent of the level of that output: having estimated that in a particular period the automobile industry must absorb ten tons of coal, say, and fifty tons of steel to make a hundred cars, we assume that it would need 0.1 tons of coal and 0.5 tons of steel per car no matter how few or how many cars it might be called upon to produce in that period. This is sometimes called the "assumption of fixed coefficients". The word "fixed" in this case should be taken to mean fixed only in relation to the scale of output, not fixed through time. The coefficients for the automobile industry in 1977, for instance, are assumed to stay the same whatever the number of cars produced in that year, but not necessarily the same as these for 1967 or 1972.

These assumptions enable us to introduce some sort of order into the bewildering variety of the real world and reduce it to measurable proportions.

On the basis of the three assumptions made above we can develop a simple input-output model as follows:

Let's go back to equation (7) of my earlier work[1/] and reproduce it here as our first equation

$$1: \quad X_i = \sum_{j=1}^{n} X_{ij} + Y_i$$

Suppose now we have an economy of five sectors (agriculture, mining and construction, manufacturing, services and government). Then if we want to

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[1/] "A note on the Relation Between Input-Output and Income and Product Accounting Systems", Unidad de Investigaciones Económicas, Serie de Ensayos y Monografías, Núm. 36. Ciencias Sociales, U.P.R., febrero, 1984. p.21.

know how much of the total output produced by the five sectors is left for final consumption then equation (1) has to be rearranged in the following way:

$$\text{1-A.} \quad X_i - \sum_{j=1}^{n} X_{ij} = Y_i$$

For the first industry of our example equation 1-A will be specified as:

$$\text{1-B.} \quad X_1 - X_{11} - X_{12} - X_{13} - X_{14} - X_{15} = Y_1$$

Or in numerical form, for agricultural sector (and expressing data in million dollars - see Table 1):

$$\text{1-C.} \quad 497.8 - 6.5 - 3.5 - 336.0 - 8.7 - 1.3 = 141.8$$

In other words the equations tell us that production minus that part sold to intermediate users is equal to that part of production left for final users.

If the amount of industry i's output purchased by each of the purchasing sectors (1, 2, 3, 4 and 5) is a stable function of latter's output we then may rewrite equation (1) as:

$$X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = Y_1$$

where:

$$\text{2.} \quad a_{11} = \frac{X_{11}}{X_1} \quad a_{13} = \frac{X_{13}}{X_3} \quad a_{15} = \frac{X_{15}}{X_5} \quad a_{12} = \frac{X_{12}}{X_2} \quad a_{14} = \frac{X_{14}}{X_4}$$

Let's define a_{ij} as matrix A. The A's are called direct input coefficients, and in an n-sector model they represent the direct requirements of the output of any sector i per unit of output of any other purchasing sector j $(i, j = 1, 2, 3, \ldots, n)$. The crucial assumption for equation (2) to hold is that the money value of goods and services delivered by an industry i to other producing sectors is a linear and homogeneous function of the output level of the purchasing sectors j. More

Table 1.

FIVE SECTORS TRANSACTION MATRIX, INTERMEDIATE IMPORTS, VALUE ADDED AND FINAL DEMAND, PUERTO RICO 1977 (in Thousand Dollars)

	Agriculture	Mining and Construction	Manufacturing	Services	Government	Total Inter-mediate Demand	Final Demand	Total Output
Agriculture	6552	3527	335970	8658	1325	356032	141822	497854
Mining and Construction	1027	13111	51776	220413	6086	292412	982517	1274929
Manufacturing	114227	429526	2943440	863915	175525	4526633	6557244	11083877
Services	38553	191805	1102997	1840040	291665	3465060	4927911	8392971
Government	22714	713	25436	60353	4371	113587	2076077	2189664
Total Domestic Intermediate Inputs	183073	638682	4434183	2993379	478972			
Intermediate Imports	51874	254630	3301013	431309	60719			
Value Added	262902	381617	3323245	4968283	1649973			
Total Expenditures	497854	1274929	11083877	8392971	2189664			

precisely, the specific assumptions are as follows: no joint products, since each commodity is supplied by a single industry and via one method of production; the linear input functions assumption means constant returns to scale and no substitutions between inputs; additivity, i.e. the total effect of production is the sum of the separate effects, which rules out external economies and diseconomies; the system is in equilibrium at given prices; and in static versions of the input-output model, no capacity constraints so that the supply of each good is perfectly elastic, thereby ignoring problems of capital.

Using equation (2) and Table 1 we are now ready to construct the direct coefficient matrix (or the so called "technological matrix") which is illustrated in Table 2[1/].

If the linear input coefficients remain constant over time, they provide a nexus for linking final demand to gross output. Input-output analysis describes the interaction of three elements of an economic system: final demands, the input requirements of each industry, and their gross output. The main analytical purpose of input-output is to determine the effects of specified changes in final demand upon gross output, given the input coefficient matrix. Such effects include not merely the direct impact, i.e. the first round of input

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[1/] In table 2 we have included intermediate imports and value added. Therefore the sums of each of the 5 columns are all equal to 1. However, the interindustrial domestic matrix is an square matrix (5 x 5) which exclude intermediate imports and factor payments (value added).

Table 2

DIRECT COEFFICIENT MATRIX PUERTO RICO, 1977

Sectors	Agriculture	Mining and Construction	Manufacturing	Services	Government
Agriculture	0.013160	0.002766	0.030312	0.001032	0.000605
Mining and Construction	0.002063	0.010284	0.004671	0.026262	0.002779
Manufacturing	0.229439	0.336902	0.265560	0.102933	0.080161
Services	0.077438	0.150444	0.099514	0.219236	0.133201
Government	0.045624	0.000559	0.002295	0.007191	0.001996
Intermediate Imports	0.104195	0.199721	0.297821	0.051389	0.027730
Value Added	0.528080	0.299324	0.299827	0.591958	0.753528
Total	1.000000	1.000000	1.000000	1.000000	1.000000

Source: Table 1

requirements, but as we shall see later, also the indirect effects of additional deliveries of these inputs on all industries in the economy.

Let's take the first row of table 2 and apply equation 2.

3.

$$\frac{X_{11}}{X_1} = 0.013160 = a_{11}$$

$$\frac{X_{12}}{X_2} = 0.002766 = a_{12} \qquad \frac{X_{14}}{X_4} = 0.001032 = a_{14}$$

$$\frac{X_{13}}{X_3} = 0.030312 = a_{13} \qquad \frac{X_{15}}{X_5} = 0.000605 = a_{15}$$

The reader can immediately observe that system (2) or (3) can be rewritten as:

4. $$X_1 - a_{11}X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = Y_1$$

or alternatively:

4-A. $$(1 - a_{11})X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = Y_1$$

If we apply the same procedure for the remaining four sectors we will obtain the following system:

5.

$$(1 - a_{11})X_1 - a_{12}X_2 - a_{13}X_3 - a_{14}X_4 - a_{15}X_5 = Y_1$$

$$-a_{21}X_1(1 - a_{22}X_2) - a_{23}X_3 - a_{24}X_4 - a_{25}X_5 = Y_2$$

$$-a_{31}X_1 - a_{32}X_2(1 - a_{33}X_3) - a_{34}X_4 - a_{35}X_5 = Y_3$$

$$-a_{41}X_1 - a_{42}X_2 - a_{43}X_3(1 - a_{44}X_4) - a_{45}X_5 = Y_4$$

$$-a_{51}X_1 - a_{52}X_2 - a_{53}X_3 - a_{54}X_4(1 - a_{55})X_5 = Y_5$$

Using the rules of matrix algebra we can express system of equations (5) as follows:

5-A.
$$\begin{bmatrix} (1-a_{11}) & -a_{12} & -a_{13} & -a_{14} & -a_{15} \\ -a_{21} & (1-a_{22}) & -a_{23} & -a_{24} & -a_{25} \\ -a_{31} & -a_{32} & (1-a_{33}) & -a_{34} & -a_{35} \\ -a_{41} & -a_{42} & -a_{43} & (1-a_{44}) & -a_{45} \\ -a_{51} & -a_{52} & -a_{53} & -a_{54} & (1-a_{55}) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_5 \end{bmatrix}$$

System number 5-A can also be presented numerically as in Table 3 or System 5-B below:

5-B.
$$\begin{bmatrix} (1-0.013160) & -0.002766 & -0.030312 & -0.001032 & -0.000605 \\ -0.002063 & (1-0.010284) & -0.004671 & -0.026262 & -0.002779 \\ -0.229439 & -0.336902 & (1-0.265560) & -0.102933 & -0.080161 \\ -0.077438 & -0.150444 & -0.099514 & (1-0.219236) & -0.133201 \\ -0.045624 & -0.000559 & -0.002295 & -0.007191 & (1-0.001996) \end{bmatrix} \times \begin{bmatrix} 497.8 \\ 1274.9 \\ 11803.9 \\ 8393.0 \\ 2189.7 \end{bmatrix} = \begin{bmatrix} 141.8 \\ 982.5 \\ 6557.3 \\ 4985.7 \\ 2018.3 \end{bmatrix}$$

From the economic meaning of the coefficients, it follows of course, that:

$$Aij \geqq 0, \qquad i,j, = 1, 2, 3, 4, 5$$

That is, the coefficients of production and of consumption are all positive or, at least, equal to zero. Negative coefficients would have no economic meaning.

Using matrix notation system (5-B) can be represented by the following equatio

6. $(I - A)\ X = F$

Where I is known as the identity matrix, which can be represented as follows:

7.
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Table 3

NUMERICAL ILLUSTRATION OF EQUATION SYSTEM NUMBER FIVE[1/]
(FIRST FUNDAMENTAL RELATION OF THE INPUT-OUTPUT MODEL)

Industrial Sectors	Agriculture	Mining and Construction	Manufacturing	Services	Government		Production		Final Demand
Agriculture	0.98684	-0.00277	-0.03031	-0.00040	-0.00276		497.8		14.8
Mining and Construction	-0.00206	0.98972	-0.00467	-0.02706	-0.00250		1274.9		982.5
Manufacturing	-0.22944	-0.33690	0.73444	-0.08963	-0.12695	X	11083.9	=	6557.3
Services	-0.07054	-0.13636	-0.08749	0.80359	-0.15693		8393.0		4985.7
Government	-0.05252	-0.01465	-0.01431	-0.02371	0.00144		2189.7		2018.3

Source: Table 1, Table 2 and equations 5-B

1/ The Direct coefficient matrix has to be converted to this format in order to be inverted.

The A's are equal to the direct coefficients, X the production and Y the final demand.

The system represented (by equation 6) has been called the Leontief system (or scheme or model) open with respect to final demand, or more simply "Open Leontief System". Under the condition that (I-A) has an inverse (in practical circumstances this condition will be met if the vector of final demand, Y, contains at least one non-zero element) we can solve the system by expressing gross output as a function of the exogenous final demand[1/]. An example taken from elementary algebra can illustrate the mechanic of the process to derive an inverse.

Suppose

$(I - A)X = Y$ can be numerically represented

by $(1 - 0.30)80 = 56$

We want to solve the system for the variable output (X = 80). Therefore, the solution is given by

$$80 = \left(\frac{1}{1-0.30}\right) 56$$

or alternatively

$$80 = (1 - 0.30)^{-1}\ 56$$

In linear algebra the solution is more complex. However, the example given above will give the reader an idea of the process involved in the solution

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[1/] In other words Leontief's analytical system pose the following question: How much output has to be produced by the different sectors of an economy in order to satisfy the final demand of consumers? (given the technology represented by the input-output coefficients).

The solution to equation (6) then is given by the following fundamental Leontief's equation:

8. $(I - A)^{-1}F = X$

where $(I - A)^{-1}$ is the so called "Leontief inverse matrix" or a direct plus indirect requirements matrix[1/].

Let $B = (I - A)^{-1}$ then:

$$9. \quad \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} \\ b_{41} & b_{42} & b_{43} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{53} & b_{54} & b_{55} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}$$

In matrix notation system (9) can be expressed as:

10. $X = BF$

Each entry in the inverse matrix is called an interdependency coefficient. The coefficient bij represents the direct and indirect requirements of sector i per unit of final demand for the output of sector j. Thus:

11. $Xi = bi_1 F_1 + bi_2 F_2 + \ldots + bij F_j + \ldots + b_{in}F_n$

The Leontief's inverse is a very powerful analytical instrument. We can multiply it by any size or composition of final demand (or its components)

[1/] Mathematically the derivation of an inverse matrix is explained by means of determinants. There will be an inverse if the determinant of the original matrix is different from 0. The inverse can also be explained by the following Serie: $\beta = I + A + A^2 + A^3 + \ldots + A^n$

to obtain the outputs by industrial sectors. Knowing other technical relations like capital-output, labor-output, energy-output, etc. coefficients we can also obtain the total requirements of these resources per unit of final demand (total means direct plus indirect requirements)[1/].

Table 4 shows the direct plus indirect coefficients matrix of 5 sectors (for convenience it also includes intermediate imports and value added).

It is worth observing that the production of the different industries (X_1, X_2 .... X_n) will clearly be greater than, or at least equal to, the amounts of final demands (Y_1, Y_2, ..., Y_n) since they must provide for the latter (for consumption and investment) and for intermediate requirements ($X_1 - Y_1$, $X_2 - Y_2$, ..., $X_n - Y_n$). From this we can infer the economic meaning of each element bij of the inverse $(I-A)^{-1}$, by contrast with the economic meaning of each element Aij of the original matrix A (direct coefficient matrix-- see Table 2). We know, of course, that each element Aij of A represents the amount of ith commodity needed in the jth industry for the production of one unit of the jth commodity. Thus the Aij coefficients may be said to represent the direct requirements of commodities for the production of commodities. In $(I-A)^{-1}$ by contrast, each element bij represents the amount of ith commodity needed in the economic system as a whole in order to obtain eventually the availability of one level of the jth commodity as a final good. The bij thus represent the total requirements (of the direct and indirect requirements) of commodities for the production of final commodities (i.e. of consumption and new investment goods).

[1/] One of the most desirable and useful features of the input-output model is its capability of showing the indirect effects of economic activity.

Table 4

MATRIX OF DIRECT PLUS INDIRECT INPUT
REQUIREMENTS (INCLUDING IMPORTS AND VALUE ADDED)
PUERTO RICO, 1977

	Agriculture	Mining and Cons- truction	Manu- facturing	Services	Government
riculture	1.024330	0.018842	0.043464	0.007765	0.005201
ning and Construction	0.008073	1.019868	0.011716	0.035938	0.008583
nufacturing	0.350917	0.511257	1.407473	0.204524	0.141983
rvices	0.156197	0.264318	0.187082	1.316320	0.191543
vernment	0.048764	0.004513	0.006578	0.010330	1.003946
tal Domestic	1.588281	1.818798	1.656313	1.574877	1.351256
ports	0.222232	0.371624	0.435840	0.136829	0.02224
lue Added	0.777767	0.628375	0.564159	0.863170	0.917775
tal	2.58828	2.81880	2.656312	2.574876	2.291255

The following example will show the practical application of the inverse matrix. Suppose we want to know the production of agriculture given the final demand of Table 1. Applying equation 11 and first row of Table 4 we obtain:

12. $497.8 = 141.8(1.02433) + 982.5(0.018842) + 6557.3(0.043464)$
$+4985.7(0.007765) + 2018.3(0.005201)$

or in symbols:

12-A. $X_1 = F_1(b_{11}) + F_2(b_{12}) + F_3(b_{13}) + F_4(b_{14}) + F_5(b_{15})$

The sum of each column corresponding to the domestic production (excluding imports and value added) is called the "production multiplier". It tells you that if the final demand for the agricultural production is 1.0 million dollars the economic system has to produce $1,588,281 in order to satisfy the above mentioned demand.

The usual method for calculating the inverse matrix is that of determinants. However, the special properties of the Leontief's inverse allow us to use an iterative method of computation[1/].

From linear algebra it is known that the square matrix:

$$\mu^{-1}(\mu I-A)^{-1}$$

Can be expanded as a convergent powers series:

$$I + \frac{1}{\mu} A + (\frac{1}{\mu} A)^2 + (\frac{1}{\mu} A)^3 + \ldots$$

provided that $\mu > |\lambda_m|$ where λ_m is the eigenvalue of A of greatest modulus. Since the Leontief's inverse is special case of the matrix $\mu^{-1}(\mu I-A)^{-1}$ where $\mu = 1$ then it can be written as the power series expansion:

13. $(I-A)^{-1} = I + A + A^2 + A^3 + \ldots.$

[1/] This part has been based on Chapter 4 and Mathematical appendixes of Luigi L. Pasinetti's book Lecturer and the Theory of Production, Columbia University Press, New York, 1977.

Provided that the eigenvalue of A with greatest modulus is in modulus less than one. Since A is a non-negative matrix it largest eigenvalue will be its eigenvalue of greatest modulus[1/].

The Price System

Input-output theory starts out from an input-output table where all elements are measured in physical units. As we specified before, this requires that each industry produces only one, well-defined commodity or service, and that the primary inputs be homogeneous too. The open input-output system expressed entirely in physical units is really a technocratic system, rather than an economic system. Neither the physical input-output table nor the I-O model deals explicitly with prices and seemingly has nothing at all to do with price determination. However, such is not the case. The system does, in fact, determine a set of equilibrium prices or rather, relative equilibrium prices.

Analogous to the relationship between Y and X, above, is the relation between prices and value added (or income generated). Thus, consider the ith column of the coefficient matrix, A. If we multiply this column by the per unit prices (given exogeneously) of the corresponding commodities we obtain a column giving the value of each input used to produce one unit of the jth good. Summing over all the commodities, we get the material cost per unit output of j. The difference between the value of one unit of j and the cost per unit of j, obtained as above, gives us the gross value added per unit of j. Thus, given the prices we could easily compute the value added per unit of output of each industry with the help of the coefficient matrix A. If however, we have value added as our datum

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[1/] The greatest eigenvalue of A must be less than one if A is to have an economic meaning.

we could easily reverse the problem and obtain prices. In matrix notation the value added and price equation are:

14. $V = P\ (I - A)$

15. $P = V\ (I - A)^{-1}$

When V gives the per unit gross value added in the ith industry.

The total value added (or income generated) in a given industry could be divided into two components of wages and profits. In such a situation the price formation equation 19 can be expressed as:

16. $P = (W + g)(I - A)^{-1}$

where (W) and (g) are vectors of wages per unit of output in each industry and profits per unit of output in each industry, respectively[1/].

Limitations of the Input-Output Model

After having explained the significance of the Leontief model, it is appropriate also to mention its limitations. These limitations may all be traced back to the basic assumption which was adopted in its construction, namely, the assumption that the technical coefficients are constant. In practice, changes in the technical coefficients can arise from two quite distinct sources. The first one is that the returns to scale may be increasing or decreasing. When this is the case, the technical coefficients are no longer independent of the scale of production. The extend to which the Leontief system might nevertheless be useful, at least as an approximation, will of course, depend on the degree of nonlinearity of the production processes. A linear relation is, after all, a good approximation to any function, provided that the variations around the point under consideration are small[2/].

1/ W and g refer to total wages and profits per unit of output of the industry and should not be confused for wages rates and profit rates.

2/ Mathematically this is a remarkable and well-known analytical result derived from the Taylor series expansion.

There is a second source of variations of the technical coefficients: Technical progress. This is more difficult to deal with. Technical progress acts upon the various coefficients quite autonomously, sometimes independently of the scale of production, and sometimes in conjunction with it. The important factor, therefore, in this respect, is time. For short periods of time, changes in technical knowledge can normally be neglected, but when periods of some years are considered technical changes can become highly relevant, and it would be unreasonable to rely on conclusions on a model in which it is assumed that the coefficient matrix remains constant.

The above system is what we have called the "open system" and for certain purposes it is very useful. However, its usefulness can be expanded if we integrate it with macro-economic accounts. When you include national accounts (like consumption, value added, imports, investment and others) as endogenous variables. Then the system becomes "closed". The solution to the close system in addition to giving the total sales by industrial sector it will also give as many macro-variables as you include as endogenous. However, the open system (or sub-system in our case) can be used independently of the augmented model to obtain value of sales by industrial sectors, impact analysis, and projections of sales.